Discrete Mathematical Structures

KR Chowdhary
Professor & Head
Email: kr.chowdhary@gmail.com

Department of Computer Science and Engineering
MBM Engineering College, Jodhpur

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Text and References

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Motivation

- Mathematical Reasoning
- Discrete Structures
- Combinatorial Analysis
- Modeling Applications
- Algorithms

What are discrete Structures?

Definition

The Discrete mathematics is branch of mathematics that deals with discrete objects.

Sets and set operations

$A = \{a, b, c, d\}$
$B = \{1, 3, 5, 7\}$
$C = \{a, 1, 2, c\}$
$a \in A$ is true
$b \notin A$ is false
$1 \in A$ is false
$\{1, 3\} \subseteq B$
$A \cup B = ?$
$A \cap B = ?$
$A - C = ?$

Set representations

- S $= \{$All students of BE CSE$\}$
- S $= \{x | x$ is student of BE IT$\}$
- S $= \{x | p(x)\}$
- P $= \{2, 4, 6, 8, 10\}$
- Q $= \{x | x$ is even number and $2 \leq x \leq 10\}$

Theorem

Proof by contradiction

Proof.

- assume that theorem is false.
- if theorem is false, then its negation is true
- Now try to prove that negation leads to contradiction to some thing already established
- Therefore, negation should be false
- Thus the theorem is true.

□

Theorem

$\forall P \ \ \phi \subseteq P$

Proof.

- assume for each set $P, \phi \subseteq P$ is false.
- Hence there is at least one set Q, for which $\phi \subsetneq Q$ is true.
- Therefore, for some $x, x \in \phi$, but $x \notin Q$. This leads to contradiction to some thing already established, i.e. $x \notin \phi$.
- Therefore,negation should be false.
- Thus the theorem is true.

□

Prove the law of distribution

Theorem

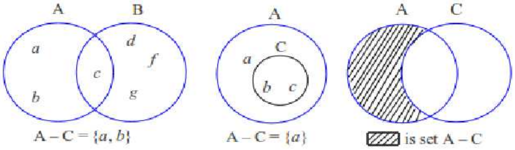
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof.

- *let* $x \in A \cap (B \cup C) \Rightarrow x \in A$ *and* $x \in (B \cup C)$
- $\Rightarrow x \in (A \cap B)$. *But* $A \cap B \subseteq (A \cap B) \cup (A \cap C)$
- *Alternatively* $x \in (A \cap C)$. *But* $A \cap C \subseteq (A \cap B) \cup (A \cap C)$
- $\Rightarrow A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$
- *let* $x \in (A \cap B) \cup (A \cap C)$
- $(A \cap B) \subseteq A \cap (B \cup C)$
- $(A \cap C) \subseteq A \cap (B \cup C)$
- $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$
- *Hence,* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

□

Set operations on Venn Diagrams



- $A - B = \{a, b\}$
- $B - A = \{d, g, f\}$
- $A \cup B = \{a, b, c, d, f, g\}$
- $A \cap B = \{c\}$
- $A \oplus B = (A \cup B) - (A \cap B)$

De Morgan's Laws

The negation of the **conjunction/disjunction** of two statements is logically equivalent to the **disjunction/conjunction** of the negation of those statements.

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$