Random TM

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TM Model Using Randomization

- Standard TM is deterministic, and for every state, tape symbol pair there is definite transition. The idea of random TM comes from random numbers theory/probability theory for solution of problems. For example, forecasting results, or accidents, or failure rates, Monte-Carlo method for integration, death and birth rates, are some of the problems solved using random/probability theory. This idea was an inspiration for random TM.
- The structure of Random TM is as follows:

- tape 1 holds input w.
- Tape 2 has random 0s and 1s. (called random tape)
- Tape 3 and subsequent tapes (if used) are initially blank and used as scratch pads.
- For random bits an internal coin flips and writes 0 or 1 (to always generate a random number before it is needed. thus, tape 2, is assumed to comprise infinite random numbers: 0,1 sequences).

Randomized version of quick sort

In quick sort, a pivot needs to be decided at random position out of the list to be sorted, and this list is split into two; those elements that are $\leq$ *the pivot and those which are greater that pivot. The same process is applied recursively on each list and so on.* The algorithm for TM is as follows:

- Pick a pivot at random and divide sublists into left and right
- If sublist to be divided is $|w| = m$, $O(n \log n)$ bits are required to pick a random number 1 to m.
- put pivot on tape 3, scan sublist on tape1, copy the number no-greater to tape 4, and greater to tape 5.
- Copy tape 4, 5 back to tape 1.
- If either sublist is of length greater than 1, recursively solve it.

  The complexity for solution is $O(n \log n)$, and complexity class is RP (**Randomized polynomial** class).
- If pivot always divides elements into sub-lists of 1 and n-1, it is insertion sort, with complexity $O(n^2)$. But, random selection of pivot postion gives average case complexity of $O(n \log n)$

Classes on randomized machine

- Class RP: (randomized polynomial)
  - These are set of languages L recognized in Randomized Polynomial time on Randomized TM M.
  - If $w \notin L$, then probability that M accepts w is 0.
  - If $w \in L$, then probability that M accepts w is at least $\frac{1}{2}$.
  - If $|w| = n$, RTM halts after $T(n)$ steps.
- Class ZAPP: (Zero error, Probabilistic, polynomial)
  - It is class based on randomized TM that always halts.
  - Time is polynomial of length of Input.
  - Expected run time instead of worst case running time is used for complexity.
- Class BPP: (Bounded error probabilistic polynomial time class
  - Solvable by probabilistic TM in P time, with error probability 1/3 of all instances.