B. Tech. III Year, I Semester, 2011
End Semester Examination Solution hints
Theory of Computation

CS 340

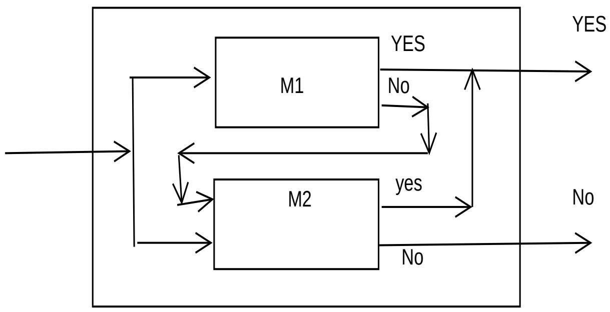
Duration: 3 Hours | Maximum marks: 50

Instructions:
i) The answers should be complete for each question. Provide the detailed steps/description, logic, and diagrams where required.
ii) All parts of each question should be attempted in continuous order.
iii) All questions carry equal marks.

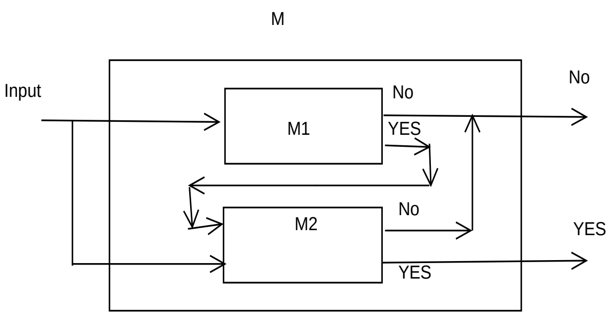
1. Prove that Recursive languages are closed on Union, Intersection, and complementation.

Ans. Let L_1 and L_2 are recursive languages, where $L_1 = L(M_1)$, $L_2 = L(M_2)$ and M_1, M_2 halt on every input. We construct the Turing machine M to simulate once $L_1 \cup L_2$, $L_1 \cap L_2$, and $\overline{L}$ (when there is TM which accepts L) one at a time.($3\frac{1}{2}, 3\frac{1}{2}, 3$ points)

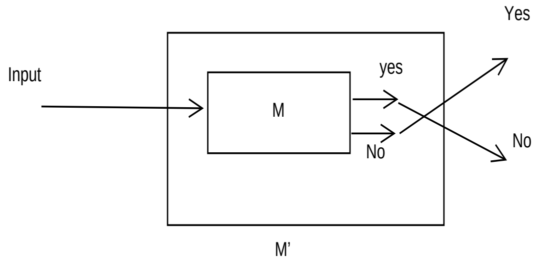
a) **Union:** The M, for $L(M) = L_1 \cup L_2$ is constructed as follows:



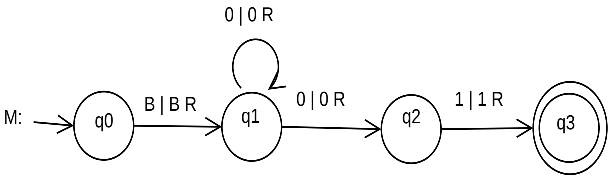
b) **Intersection:** $L(M) = L_1 \cap L_2$



c) **Complement:** We just switch the accepting and non-acepting states of M to construct the new Turing machine. Thus, $L(M') = \overline{L}$.



2. (a) Given the Turing Machine M in figure below, with input alphabets $\Sigma = \{0, 1\}$, tape alphabets $\Gamma = \{0, 1, B\}$, find out the language $L(M)$ accepted by this machine.(4 points)



Ans. For $0|0R$ at state q_1: $L(M) = 0^*01(1+0)^*$

Ans. For $1|0R$ at state q_1: $L(M) = 1^*01(1+0)^*$

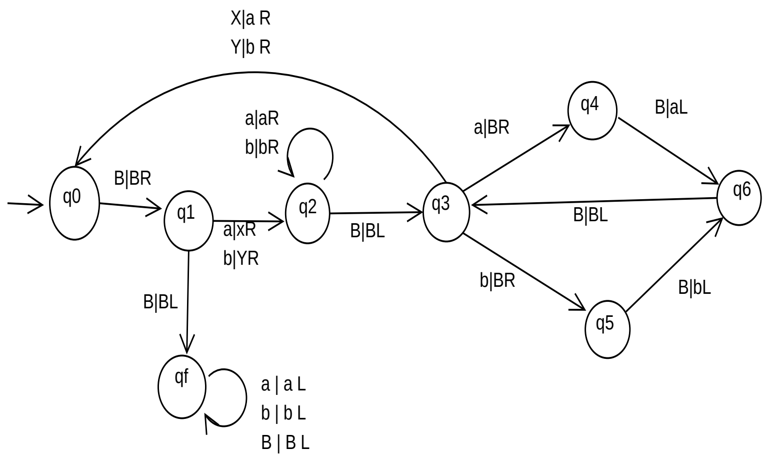
Once a TM has reaching to final accepting, further input has no meaning. Hence, the suffix $(1+0)^*$ for the language string 0^*01 or 1^*01, depending on whether $0|0R$ or $1|0R$ has been taken as the transition at q_1.

(b) Given the alphabet set $\Sigma = \{a, b\}$, construct a standard Turing machine, which inserts a blank space between each of the input symbol for any arbitrary string $w \in \Sigma^*$ available on tape. Write the algorithmic steps, details of all the tuples of machine, and give the transition diagram.(6 points)

Ans. 1. The R/W head goes marks the left most unmarked char a by X or b by y, goes to right most position, moves rightmost char one position write, creating space it that position. And, moves this space left till x / y postion.

2. In similar way, by first shifting the right most char on one position, the blank creating is shifted left, till the previoussly inserted space.

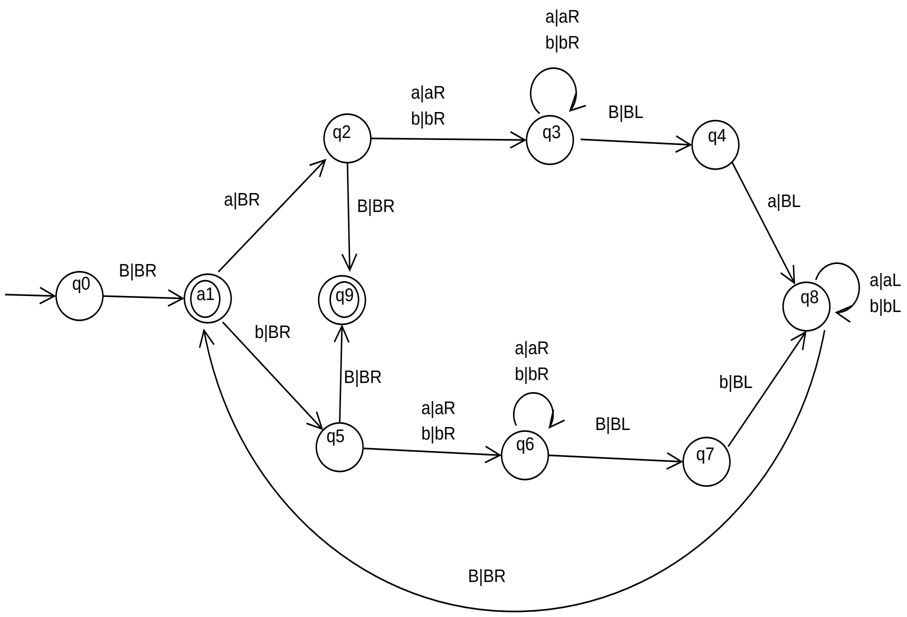
3. Scaning R/W head, right and left, it insert space inbetween call characters.



3. Given L as set of palindromes over $\Sigma = \{a, b\}$,

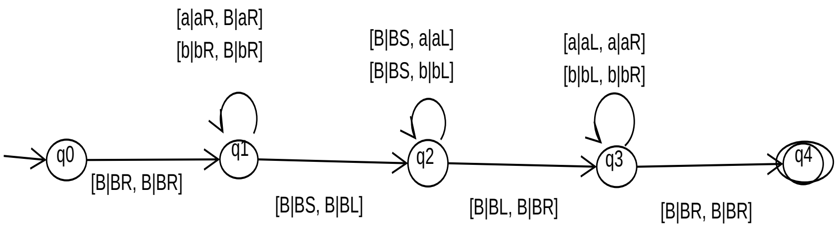
   a) Build standard Turing machine that accepts L.(5 points)

Ans. The below given standard TM accepts the languages of odd and even palindromes over $\Sigma = \{a, b\}$



   b) Build a two-tape Truing machine that accepts L, which takes computation time no more than $3len(w) + 4$ transitions, where w is input, and $len(w)$ is length of input. (6 points)

Ans. The transition diagram of two-TM is given below. Total time for $|w| = n$ is $3n + 4$, where $3n$ is due to three loops,and constant 4 is due to four non-loop transitions.



4. (a) Represent the relation between $P, NP, NP - complete$ and $NP - hard$ problems using Venn-Diagram, and justify this relationship.(5 points)

   Ans. The figure of relations is shown below.

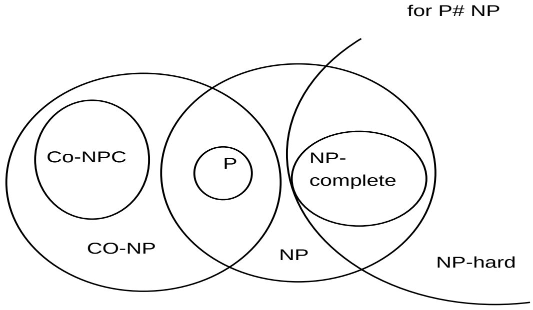
   (b) Classify the following problems into $P, NP, NP - complete$ and $NP - hard$, giving a brief logical justification(5 points).

      i. Solving Towers of Hanoi problem,

   Ans. NP.

      ii. Solving Traveling salesman problem,

   Ans. NP-Complete. Because, $3SAT <_P TSP$



iii. Finding whether there exists a Hamiltonian path in an undirected graph $G = (V, E)$. (A Hamiltonian path is a path connecting all the vertices in the graph, without repeating any).

Ans. NP-Complete. Because $3SAT <_P HamiltonianPP$

iv. Solving Subgroup sum problem; given $S = \{a_1, a_2, \ldots, a_n\}$, whether a_i are non-negative integer number, and there exists a subgroup of S, which has sum equal to a given integer number m.

Ans. NP-complete. Because, $3SAT <_P subsetsum$

5. Make use of Halting problem of Turing machine, and show that the "general case of testing a tester program P by itself", is unsolvable; the program P is designed to test other programs. (10 points)

Ans. (solution Hint) The case is same as the solution of Halting problem. In the halting problem we take input as $\langle M\rangle\langle M\rangle$, here the input is $\langle P\rangle\langle P\rangle$.