

Random TM

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TM Model Using Randomization

- Standard TM is deterministic, and for every state, tape symbol pair there is definite transition. The idea of random TM comes from random numbers theory/probability theory for solution of problems. For example, forecasting results, or accidents, or failure rates, Monte-Carlo method for integration, death and birth rates, are some of the problems solved using random/probability theory. This idea was an inspiration for random TM.
- The structure of Random TM is as follows:
 - tape 1 holds input w .
 - Tape 2 has random 0s and 1s. (called random tape)
 - Tape 3 and subsequent tapes (if used) are initially blank and used as scratch pads.
 - For random bits an internal coin flips and writes 0 or 1.

Randomized version of quick sort

In quick sort, a pivot needs to be decided at random out of the list to be sorted, and this list is split into two; those elements that are \leq the pivot and those which are greater than that pivot. The same process is applied recursively on each list and so on. The algorithm for TM is as follows:

- Pick a pivot at random and divide sublists into left and right
- If sublist to be divided is $|w| = m$, $O(n \log n)$ bits are required to pick a random number 1 to m .
- put pivot on tape 3, scan sublist on tape 1, copy the number no-greater to tape 4, and greater to tape 5.
- Copy tape 4, 5 back to tape 1.
- If either sublist is greater than 1, recursively solve it.

The complexity for solution is $O(n \log n)$, and complexity class is RP (Randomized polynomial class).

Classes on randomized machine

- Class RP: (randomized polynomial)
 - These are set of languages L recognized in Randomized Polynomial time on Randomized TM M .
 - If $w \notin L$, then probability that M accepts w is 0.
 - If $w \in L$, then probability that M accepts w is $\frac{1}{2}$.
 - If $|w| = n$, RTM halts after $T(n)$ steps.
- Class ZAPP: (Zero error, Probabilistic, polynomial)
 - It is class based on randomized TM that always halts.
 - Time is polynomial of length of Input.
 - Expected run time instead of worst case running time is used for complexity.
- Class BPP: (Bounded error probabilistic polynomial time class)
 - Solvable by probabilistic TM in P time, with error probability $1/3$ of all instances.