Turing Machines Extensions

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Ways to Extend Turing Machines

Many variations have been proposed:

- Allow two-way infinite tape
- Allow multiple tapes
- Allow Multiple heads
- Allow two dimensional TM
- Allow a multidimensional tape
- Allow non-determinism
- Allow combinations of the above
- Theorem: The operations of a TM allowing some or all the above extensions can be simulated by a standard TM.The extensions do not give us machines more powerful than the TM.
- The extensions are helpful in designing machines to solve particular problems.

Multiple Track or multi-Tape TM

- Extensions to TM add no computation power to it:
- Two tapes, each with its own read-write head
  1. In each step, TM reads symbols scanned by all heads, depending on those and current state, each head writes, moves R, L, and control unit enter new state.
  2. Actions of heads are independent of each other
  3. Tape position in two tracks: $[x, y]$, x in first track, and y in second. δ is given by: $\delta(q_i, [x, y]) = (q_j, [z, w], d, d), \;\; d \in \{L, R\}$

δ	a, a	$\#, a$	$a, \#$	$\#, \#$
q_0	q_1, a, b, L, R	$q_2, b, \#, L, L$	$q_0, b, \#, L, R,$	$\ldots$

- Example: Copying string from one tape to another tape?

Multiple Track or multi-Tape TM

for multitape: k number of tapes

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R\}^k$$

$$\delta(q_i, a_1, \ldots, a_k) = (q_j, (b_1, \ldots, b_k), (d_1, \ldots, d_k))$$

Multiple Track or multiTape TM

How can a standard TM simulate a three-tape TM?

- Interleave contents of two tapes onto a single tape.

  01010# Tape 1, red color is head position

  aaa# Tape 2

  ba# Tape 3

- Tape contents on simulated standard TM, $*$ is separator:

  01010 $*$ aaa $*$ ba# Single Tape contents

- In practice, S leaves an extra blank before each symbol to record position of read-write heads

Simulate Multiple Track on standard TM

How can a standard TM simulate a three-tape TM?...

1. S reads the symbols under the virtual heads (L to R).
2. Then S makes a second pass to update the tapes according to the way the $M's$ transition function dictates.
3. If, at any point S moves one of the virtual heads to the right of $*$, it implies that head moved to unread blank portion of that tape. So S writes a blank symbol in the right most of that tape.Then continues to simulate.

   $\Rightarrow$ control will need a lot more states.

Multitape Turing Machine = Single tape TM

Theorem

A language is accepted by a two tape TM if and only if it is accepted by a standard TM.

Proof.

- *Part 1:If L is accepted by standard TM then it is accepted by two tape TM also(simply ignore 2nd tape), i.e.,*$[a, \#]$.
  Part 2:
- *let* $M = (Q, \Sigma, \Gamma, \delta, q_0, H)$ *be two track. Find one track equivalent?*
- *Create ordered pair* $[x, y]$ *on single tape machine* M'.
- $M' = (Q, \Sigma \times \{\#\}, \Gamma \times \Gamma, \delta', q_0, F)$ *with* δ' *as* $\delta'(q_i, [x, y]) = \delta(q_i, [x, y])$.

Nondeterministic TM

- Has finite number of choices of moves; components are same as standard TM; may have > 1 move with same input $(Q \times \Sigma)$. Nondeterminism is like FA and PDA.
- Example: Find if a graph has a connected subgraph of k nodes (no efficient algorithm exists).Non-exhaustive based solution is Guess & check.
  1. NDTM: Arbitrarily choose a move when more than one possibility exists for $\delta(q_i, a)$.
  2. Accept the input if there is at least one computation that leads to accepting state (however, the converse is irrelevant).
- To find a NDTM for ww input, $w \in \Sigma^*$, you need to guess the mid point. A *NDTM* may specify any number of transitions for a given configuration, i.e. $\delta : (Q - H) \times \Gamma \rightarrow subset\ \ of\ \ Q \times \Gamma \times \{L, R\}$

Nondeterministic TM ...

Example: $w = ucv$, where c is preceded by or followed by ab

Approach: Read input a, b, c and write a, b, c respectively, and move R in each, at start state. Then with input c, Nondeterministically decide c, a, b by moving R in three states transitions or decide c, b, a by moving L in three other states transitions (i.e., abc)

Transformation of NDTM to Standard TM

- A NDTM produces multiple computations for a single string. We show that multiple computations $m_1, \ldots, m_i, \ldots, m_k$ for a single input string can be sequentially generated and applied.
- These computations can be systematically produced by adding the alternative transitions for each $Q \times \Sigma$ pair. Each m_i has number of transitions $1 - n$. If $\delta(q_i, x) = 0$, the TM halts.
- Using the ability to sequentially produce the computations, a *NDTM* M can be simulated by a 3-tape TM M'.

Transformation of NDTM to Standard TM ...

Every nondeterministic TM has an equivalent 3-tape Turing machine, which, in turn, has an equivalent 1-tape Turing machine.

Simulation of a NDTM by 3-tape TM

- Tape-1 stores the input string, tape-2 simulates the tape of M, and tape-3 holds sequence $m_1, \ldots, m_i, \ldots, m_k$ to guide the simulation.
- Computation of $M^{'}$ consists following:
  - A sequence of inputs $m_1, \ldots, m_i, \ldots, m_k$, where each $i = 1, n$ is written on tape-3.
  - Input string is copied on tape-2.
  - Computation of M defined by sequence on tape-3 is simulated on tape-2.
  - If simulation halts prier to executing k transitions, computations of$M^{'}$ halts and accepts input, else
  - The Next sequence is generated on tape-3 and computation continues on tape-2.

Two-way infinite tape

- There is single tape which extends from $-\infty$ *to* $+\infty$. One R-W head, $M = (Q, \Sigma, \delta, q_0, F)$

  . . . -3 -2 -1 0 1 2 3 . . . , is square sequence on TM, with R-W head at 0

  This can be simulated by a two-track TM:

- $M^{'} = (Q^{'} \cup \{q_s, q_t\}) \times \{U, D\}$, where U $=$ up tape head, D $=$ down tape head, $\Sigma^{'} = \Sigma, \Gamma^{'} = \Gamma \cup \{\#\}$, and $F^{'} = \{[q_i, U], [q_i, D] | q_i \in F\}$. Initial state of $M^{'}$ is pair $[q_s, D]$. A transition from this writes $\#$ in U track at left most position. Transition from $[q_t, D]$ returns the tape head to its original position to begin simulation of M.

Multi-Dimensional Tape:

- Single R-W head, but multiple tapes exists. Let the Dimensions be 2D. For each input symbol and state, this writes a symbols at current head position, moves to a new state, and R-W head moves to left or right or up or down.

  Simulate it on 2-tape TM:

- copy each row of 2-D tape on 2nd tape of 2-tape TM. When 2D TM moves head L or R, move the head on 2nd-tape of two-tape also L or R. When 2D head moves up, 2nd tape of two-tape scans left until it finds $*$. As it scans, it writes the symbols on tape-1. Then scans and puts remaining symbols on tape-1. Now it simulates this row (on tape-1).