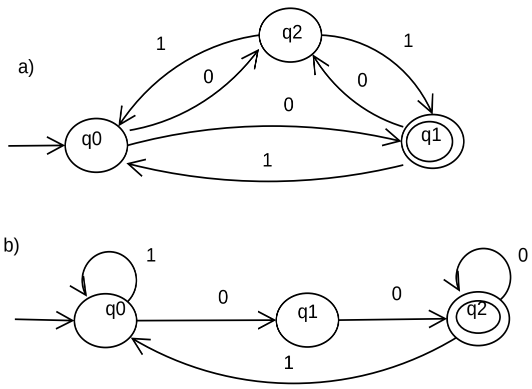
BE CSE/IT V Sem. 2011-12
Theory of Computation

Home work sheet #1

December 16, 2011

1. Describe concisely the strings recognized by the following FA:



2. Give DFA's accepting the following languages over $\{0, 1\}$.
   (a) All strings that begin and end with 00 or 11.
   (b) Before every pair of 0´s there is at least one 1.
   (c) Set of all strings such that each block of five consecutive symbols contain at least two 0´s.
   (d) Set of all the strings that either begin or end (or both) with 01.
   (e) The set of strings such that no. of 0´s are divisible by five, and number of 1´s are divisible by three.
   (f) The set of all strings beginning with 1, that when interpreted as a binary, is multiple of 5. For example, strings, 101, 1010, 111, are all in language, and 0, 100, 111 are not.
   (g) Set of all strings that, when interpreted *in reverse* as binary integer, is divisible by 5. For example, $L = \{0, 101, 0101, 1111, 00101, \dots\}$.
3. Write regular expressions for the following languages:
   (a) The set of strings over alphabet, $\{a, b, c\}$ containing at least one a and at least one b.
   (b) The set of strings of $0, s$ and 1´s whose 10^{th} symbol from right is 1.
   (c) Set of strings over $\{0, 1\}$, with at most one pair of consecutive 1´s.
   (d) No. of 0´s is divisible by five.
   (e) Not containing 101 as substring.
   (f) No. of 0´s is divisible by 5 and whose number of 1´s is even.
4. Give the description of following regular expressions, as what type of languages they are generating:

(a) $(1+\varepsilon)(00^*1)^*0^*$
(b) $(0^*1^*)^*000(0+1)^*$
(c) $(0+10)^*1^*$

5. Construct the NFA with ε-transitions for the followings:
   (a) 01^*
   (b) $(0+1)^*01$
   (c) $00(0+1)^*$
6. Discuss the applications of regular expressions.
7. Prove that regular languages are closed on followings:
   (a) Complement
   (b) Difference
   (c) Reversal(of string)
   (d) Homomorphism (substitution of strings by symbols) of a regular is regular.
8. Show that set of strings over $\{0,1\}^*$ which are binary representation of numbers divisible by 5, is regular.
9. Use the pumping Lemma to prove that following languages are not regular:
   (a) $\{0^n10^n|n \geq 1\}$
   (b) $\{0^n1^m|n \leq m\}$
   (c) $\{ww|w \in \{a,b\}^*\}$
   (d) $\{0^n|n = 2^m \text{ and } m \in \mathbb{N}\}$
   (e) $L = \{a^lb^mc^n|l,m,n \geq 0, \text{ and } m \leq n\}$
10. Given NFA of n-states, find out the complexity of time and space for constructing equivalent DFA.
11. Suggest appropriate methods to prove that:
    (a) Given two DFA are equal
    (b) Given two regular expressions are equal.
12. Let L be a regular language over some alphabet Σ.
    (a) Is the language L_1 consisting of all strings in L of length ≤ 200 a regular language?
    (b) Is the language L_2 consisting of all strings in L of length > 200 a regular language? Justify your answer in both cases.
13. How many distinct $DFAs$ are there on a given set of n states over an alphabet with k letters ?
14. Let Σ be an alphabet, and let L_1 , L_2 , L be languages over Σ. Prove or disprove the following statements:(if false, then provide a counter example).
    (a) If $L_1 \cup L_2$ is a regular language, then either L_1 or L_2 is regular.
    (b) If L_1L_2 is a regular language, then either L_1 or L_2 is regular.
    (c) If L^* is a regular language, then L is regular.
15. Let R be any regular language. Prove that the language $L = \{w|www \in R\}$ is regular.

Note: The home work should be done in loose sheets, stapled together with name, class, branch, etc. It should be detailed, with transitions diagrams, and full-steps of proofs, where required. The description should be precise and to the points. Neat and clean work is important. The solutions must be in the order of questions. Writing question is compulsory, followed by proof/answer.
Submission deadline: Dec. 24, 2011.