Quiz # 2, B.Tech. V Sem 2011-12, IITR
Theory of Computation

Your Roll no:..

Instructions:

i. Tick the correct answer. Time: 30 minutes.

ii. Correct answer = 2 marks, wrong answer = $-\frac{1}{2}$ marks.

1. What is *Space Complexity* of multiplying x and y binary strings using standard TM? Assume that $|x| = m, |y| = n$.

   Ans. The length of x and y are m and n, hence length of $m \times n$ is $m * n$. Thus maximum space occupied any time is $m * n$, and space complexity is $O(mn)$.

2. What is *Time Complexity* of recognizing $L = \{w^R w w^R | w \in \{a,b\}^*\}$ on 3-tape TM? Write brief steps.

   Ans. Initial head positions are: $h_1 w^R w w^R$, $h_2 BB$, $h_3 BB$

   1. Copy $x = w^R w w^R$ to tape2, and tape3.(Now heads are: $w^R w w^R h_1$, $w^R w w^R h_2$, $w^R w w^R h_3$) No. of transitions=$3w$

   2. For each 3-step L move of h_1, have the one step L move of h_2, and h_3, till h_1 is extreme left. (Now heads are: $h_1 w^R w w^R$, $w^R w h_2 w^R$, $w^R w h_3 w^R$)No. of transitions=$3w$

   3. Compare: move h_1, R, h_2 L, and h_3 R, and compare. No. of transitions=w.

   if $|w^R w w^R| = n = 3|w|$, then time complexity= $n + n + \frac{1}{3}n = 2\frac{1}{3}n = O(n)$.

3. Let $G = (\{S\}, \{a,b\}, P, S)$ be a CFG where P is $S \to aSb|SS|\varepsilon$. Which of the following is true?

   (a) G is not ambiguous

   (b) There exists $x, y \in L(G)$ such that $xy \notin L(G)$.

   (c) There is deterministic PDA that accepts $L(G)$

   (d) We cannot find deterministic PDA that accepts $L(G)$.

   Ans: A is not correct because for same expression there are more than syntax trees. Because of $\varepsilon-$transition, deterministic PDA is not possible. Hence (D).

   We may take derivations: $S \Rightarrow aSb \Rightarrow aSSb \Rightarrow aaSbaSbb \Rightarrow aababb$. Can we detemine this string using deterministic PDA? NO.

4. Given TM M with $\Gamma = \{0,1,B\}$, $\Sigma = \{0,1\}$, B is for end of string, and δ is:

	Input 0	Input 1	input B
q_0	$(q_1,1,R)$	$(q_1,1,R)$	Halt
q_1	$(q_1,1,R)$	$(q_0,1,L)$	(q_0,B,L)

Which of the following is true?

(a) M cannot halt on any string $(0+1)^+$ (b) M cannot halt on any string $(00+1)^+$

(c) M halts on any string ending in 00 (d) M halts on any string ending in 1

Ans. A, B are both true.

5. Let N_f and N_p denote the classes of languages accepted by non-deterministic finite automata and non-deterministic push-down automata, respectively. Let D_f and D_p denote the classes of languages accepted by deterministic finite automata and deterministic push-down automata respectively. Which one of the following is TRUE?

(a) $D_f \subset N_f$ and $D_p \subset N_p$ (c) $D_f = N_f$ and $D_p = N_p$

(b) $D_f \subset N_f$ and $D_p = N_p$ (d) $D_f = N_f$ and $D_p \subset N_p$

Ans. $L(DFA)$ and $L(NFA)$ are equal as they recognize the same language. Some languages which are recognized by NPDA but they cannot be recognized by DPDA. Thus, L(PDA) is proper subset of L(NPDA). Ans. (D).

6. Consider the languages: $L_1 = \{a^n b^n c^m | n, m > 0\}$ and $L_2 = \{a^n b^m c^m | n, m > 0\}$. Which one of the following statements is FALSE?

(a) $L_1 \cap L_2$ is a context-free language (b) $L_1 \cup L_2$ is a context-free language

(c) L_1 and L_2 are context-free languages (d) $L_1 \cap L_2$ is recursively enumerable

Ans. C and D are false statements.

7. Consider the languages:$L_1 = \{ww^R | w \in \{0,1\}^*\}$, $L_2 = \{w\#w | w \in \{0,1\}^*\}$, where # is a special symbol, $L_3 = \{ww | w \in \{0,1\}^*\}$. Which one of the following is TRUE?

(a) L_1 is a deterministic CFL (b) L_2 is a deterministic CFL

(c) L_3 is a CFL, but not a deterministic CFL (d) L_3 is a deterministic CFL

Ans. B: dterminimistic CFL.

8. Let L_1 be a recursive language. Let L_2 and L_3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

(A) $L_2 - L_1$ is recursively enumerable (B) $L_1 - L_3$ is recursively enumerable

(C) $L_2 \cap L_1$ is recursively enumerable (D) $L_2 \cup L_1$ is recursively enumerable

Ans. C.