Universal Turing Machines

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October 8, 2010

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- Can a TM be simulated by a TM?- YES
- Can one Turing machine simulate every TM?

Universal TM

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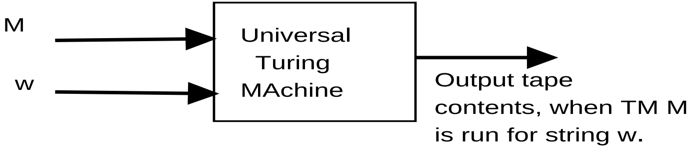
- A UTM can be designed to simulate the computations of an arbitrary TM M. To do so, input to UTM must contain representation of both - machine M and string w processed by M.



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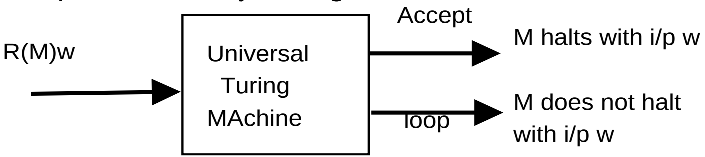
- Output1: Accept (indicates that M halts with input w), output2: loops, i.e., M does not halt with input w, i.e. computation of does not terminate
- The machine U is called universal TM, as computation of any Turing machine can be simulated by U.



Step1: design a string representation of a TM M

Because of the ability to encode arbitrary symbols as strings over $\{0, 1\}$, we consider Turing machine with inputs $\{0, 1\}$ and tape symbols $\Gamma = \{0, 1, \#\}$

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- TM M is defined by its transition function:

  $\delta(q_i, a) = (q_j, b, d); \;\; q_i, q_j \in Q; \;\; a, b \in \Gamma; \;\; d \in \{L, R\}$

Encoding of elements of M

Symbol	Encoding
0	1
1	11
#	111
q_0	1
q_1	11
...	...
q_n	1^{n+1}
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- let $en(z)$ denote the encoding of z. Hence, Transition $\delta(q_i, a) = (q_j, b, d)$ is encoded by string $en(q_i)0en(a)0en(q_j)0en(b)0en(d)$. *0* separates the components of δ.

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$\delta(q_0, \#) = (q_1, \#, R)$	101110110111011
$\delta(q_1, 0) = (q_0, 0, L)$	1101010101
$\delta(q_1, 1) = (q_2, 1, R)$	110110111011011
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Verification of representation of M:

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- TM can be constructed to check whether an arbitrary string $u \in \{0,1\}^*$ is encoding of deterministic TM M. Computations examines whether 000 is prefix, followed by finite sequences of encoded transitions are separated by 00s, then finally 000. (This is not the w to be recognized !!).

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3-tape Deterministic TM U:

- Tape-1 holds $R(M)w$. Tape-3 simulates computations of of M for input w. Tape-2 as working tape.

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i. q_i replaced by q_j on tape-2.

ii. b is written on tape 3, and tape head on tape-3 is moved for direction given in d.

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3. Encoding of q_0,i.e., 1 is written tape-2.
4. Transition of M is simulated on tape-3. The transition is determined by symbol scanned on tape-3 and state encoded on tape-2. Let these are a and q_i.
   - a Tape-1 is scanned for a and q_i as first two components of a transition. If not found, U halts by accepting input.
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     - i q_i replaced by q_j on tape-2.
     - ii b is written on tape 3, and tape head on tape-3 is moved for direction given in d.
5. Go back to step 4, and carry on computation by