

## TOC-Assignment # 1 (Topics: Preliminaries, Regex)

1. If it is given that  $x > 4$ , then show that  $2^x \geq x^2$ .
2. Prove that  $(uv)^R = v^R u^R$ , where  $u, v$  are strings on some alphabets, and  $R$  stands for reversal of string.
3. Which of the following functions are computable?
  - a.  $S = \{x \mid x \notin x\}$
  - b.  $f : x \rightarrow$  if  $odd(x)$  then print  $x$  else  $x := x + 1$ .
  - c.  $f : x \rightarrow succ(x)$ .
4. If  $f : A \rightarrow B, g : b \rightarrow C$ , then prove the following:
  - a. If  $f$  and  $g$  are both *injection*, then  $g \circ f$  is also *injection*.
  - b. If  $f$  and  $g$  are both *bijection*, then  $g \circ f$  is also *bijection*.
5. Can there be an algorithm that can find out whether a given **C** program halts on input  $x$ ? Justify your answer for Yes/No.
6. Given the languages  $L_1 = \{\varepsilon, 0, 1\}$  and  $L_2 = \{\varepsilon, 01, 11\}$ , find out  $L_1 \cup L_2, L_1 \cap L_2, L_1 \circ L_2, L_1^*, \bar{L}_1$ .
7. List any number of problems which are unsolvable, with brief description for each.
8. For  $a \in \Sigma$ , and three languages  $A, L, M$  on  $\Sigma$ , and  $n > 1$ , show that:
  - a.  $\{a\} \circ L = \{a\} \circ M \Rightarrow L = M$ .
  - b.  $A \circ L = A \circ M \not\Rightarrow L = M$
  - c.  $L^* = M^* \not\Rightarrow L = M$ .
  - d.  $L^n = m^n \Rightarrow L = M$ .
9. Which of the following languages are equal?  
 $(L \cup M)^*, (L \circ M)^* \circ L, (L^* \cup M^*)^*, (L^* \circ M^*)^*$ .
10. What is regular expression for  $\Sigma = \{0, 1\}$ , where each string in the language contains at least one 1.
11. Prove that all the finite languages are regular.
12. What are the regular expressions for following, for  $\Sigma = \{a, b\}$ .
  - a. All strings have no more than two  $a$ .

- b. All strings have even counts of  $a$  or  $b$ .
  - c. All strings are beginning and ending with  $a$  and have at least one  $b$ .
  - d. Length of all the strings are divisible by 4.
13. Find the regular expressions corresponding to the following regular sets:
- a.  $\{ab, ac, ad\}$
  - b.  $\{ad, ae, af, bd, be, bf, cd, ce, cf, \dots\}$
14. if  $r_1, r_2, r_3$  are the regular expressions, then the language corresponding to  $r_1(r_2 + r_3)$  is same as that of  $r_1r_2 + r_1r_3$ .
15. What are the regular expressions corresponding to the following?
- a. Decimal integer with or without sign.
  - b. Decimal float type number
  - c. C language variable representation.