Closure properties of Context-free languages and Gmammars

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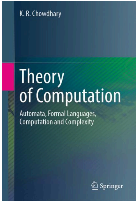
Theory of Computation

Automata, Formal Languages, Computation and Complexity

Focuses on pedagogy in its writing, that represents a refreshing approach

Ensures comprehensive and enjoyable learning

Undergone a rigorous classroom testing



Closure properties of CNF

- **Intersection of two CFLs:** Let G_1, G_2 be two context-free grammars.

G_1 :

$S \rightarrow AB$, $S \rightarrow A, A \rightarrow 0A1$

$B \rightarrow 0B$, $B \rightarrow 0$

$\therefore L(G_1) = \{0^n1^n0^+\}$

G_2 :

$S \rightarrow BA$, $S \rightarrow A$, $A \rightarrow 1A0$

$A \rightarrow 10$, $B \rightarrow 0B$, $B \rightarrow 0$

$\therefore L(G_2) = \{0^+1^n0^n\}$

$\therefore L_1 \cap L_2 = 0^n1^n0^n \notin CFL$ for $n \geq 1$.

- **Union of two CFLs:** For $L_1 = (G_1)$ and $L_2 = (G_2)$, $L_1 \cup Ł_2 \in CFL$.

  $S \rightarrow S_1|S_2$, and $V_1 \cap V_2 = \phi$

  $P = P_1 \cup P_2 \cup \{S \rightarrow S_1|S_2\}$, $V = V_1 \cup V_2 \cup \{S\}$, $\Sigma = \Sigma_1 \cup \Sigma_2$.

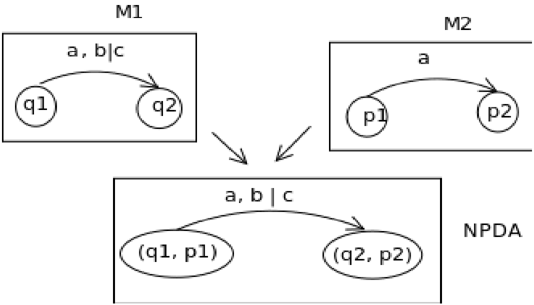
- **Concatenation of two CFLs:** For $L_1 = (G_1)$ and $L_2 = (G_2)$, $L_1 \circ Ł_2 \in CFL$.

  $S \rightarrow S_1 \circ S_2$, and $V_1 \cap V_2 = \phi$

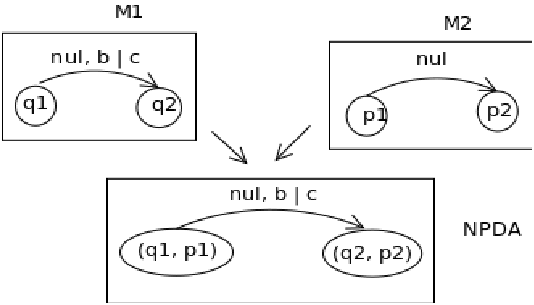
  $P = P_1 \cup P_2 \cup \{S \rightarrow S_1 \circ S_2\}$, $V = V_1 \cup V_2 \cup \{S\}$, $\Sigma = \Sigma_1 \cup \Sigma_2$.

- **Kleene star of two CFLs:** For $L_1 = (G_1)$ and $L_2 = (G_2)$, $L_1^* \in CFL$, where $S \rightarrow S_1S|\varepsilon$, $V_1 \cap V_2 = \phi$.

- **CFL** $\cap$**Reg. lang** $\in CFL$
- Let M_1 is NPDA accepting CF language L_1 by final state, and M_2 be a FA accpeting L_2. The PDA recognizing $L_1 \cap L_2$ simulates P and M simultaneously, like cross-product of two FA.
- We construct new $NPDA$ M for $L_1 \cap L_2$ to simulate M_1 and M_2 in parallel.



- **CFL $\cap$ Reg. lang** $\in CFL \ldots$



- **Simulaitng start state:** For $q_0 \in M_1, p_0 \in M_2$ there is $(q_0, p_0) \in M$
- **Simulaitng final state:** For $q_1 \in F_1$, and $p_1, p_2 \in F_2$ there is $(q_1, p_1), (q_1, p_2) \in F$.

decision problems for CFLs

- **Membership problem:** For CFG G_1, find if $w \in L(G)$?

  The membership algorithm is: Parser. That is, if we are able to obtain a parse-tree for given word w, then $w \imath L(G)$ else not.

- **Empty Language:** Is $L(G) = \phi$?

  Algorithm:

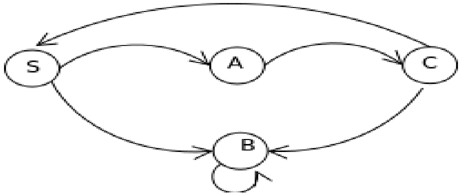
  1. Remove useless symbols
  2. Check if start symbol is useless? If yes, then $L(G) = \phi$ else not.

- **Infinite Language Problem:** Is $L = L(G)$ an infinite language?

  Algorithm:

  1. remove useless symbols
  2. remove null and unit productions
  3. create dependency graph for variables
  4. if there is a loop in the dependency graph, then L is infinite language else not.

- **Infinite Language Problem:** Is $L = L(G)$ an infinite language? ...
  Let the gramamr be:
  $S \to AB, A \to aCb|a, B \to bB|bb$
  $C \to cBS$



Since there is a loop in the dependency graph, the language is infinite. The derivation is $S \Rightarrow^* (acbb)^i S(bbb)^i$.

Bibliography

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